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Lower bounds to irregular frequencies in wave-ship interactions for engineering applications

Samon Ando

Defence R&D Canada

Technical Memorandum
DREA TM 2001-192
January 2002

National Defence Défense nationale

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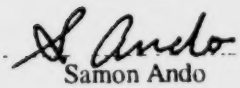

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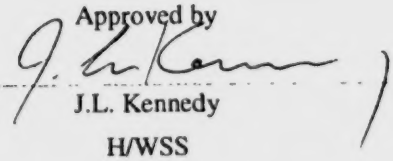

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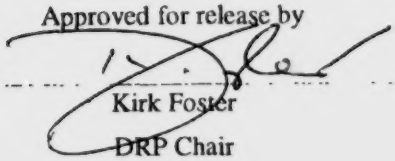

Kirk Foster

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Abstract

An analytical method has been developed to separate the frequency domain for calculations of ship motions and wave loads into two regimes. One is free of irregular frequencies and for which the well-established and well-validated ordinary integral equation method is appropriate. The other includes irregular frequencies and for which an alternative method for suppressing irregular frequencies is required. This dichotomy of the frequency range was achieved by obtaining sharp estimates of lower bounds to the irregular frequencies inherent to the integral-equation method. This work was motivated by the practical need to ensure the efficiency as well as accuracy of the calculations of ship motions and wave loads. Knowledge of two frequency regimes enables one to incorporate a "switch" in the computer codes so as to activate any one of the time-consuming, and often numerically unstable, alternative methods for suppressing irregular frequencies only when it is truly needed. In most cases, however, the need for the latter method would arise only rarely. Theoretically, the present method is based on the monotone relation between the magnitudes of the fundamental eigenvalue and the domain of the interior boundary-value problem associated with the given exterior problem for wave-body interactions, which is derived in this report.

Résumé

Nous avons élaboré une méthode analytique qui simplifie le calcul des mouvements des navires et de la charge de la houle, par la division du domaine des fréquences en deux régimes. Le premier régime n'est pas contaminé par les fréquences irrégulières et se prête aux calculs par la méthode bien établie et validée des équations intégrales. C'est dans le second que se trouvent les fréquences irrégulières qu'il faut éliminer en recourant à une autre méthode. Nous avons pu diviser le domaine des fréquences en calculant une estimation précise des limites inférieures des fréquences irrégulières qui sont un sous-produit de la méthode des équations intégrales. La recherche pragmatique d'efficacité et d'exactitude pour les calculs des mouvements des navires et de la charge de la houle a motivé ce travail. La connaissance des deux régimes de fréquence nous a permis d'ajouter au logiciel un « commutateur » qui aiguille les calculs, lorsque nécessaire, vers l'une des autres méthodes — chronophages et souvent instables — qui permettent d'éliminer les fréquences irrégulières. Dans la plupart des cas, le recours à ces méthodes est exceptionnel. Théoriquement, la méthode présentée ici repose sur la relation monotone entre les grandeurs des valeurs propres fondamentales et le domaine du problème intérieur des valeurs limites associées au problème extérieur donné des interactions houle-corps, dont le calcul est présenté dans ce rapport.

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Executive summary

The ability to predict how a ship will behave when excited by waves is essential in ensuring the safety of the ship. The major effort in developing an analytical model of ship motions is spent on the calculation of the velocity potential functions. The most convenient method for calculating the velocity potential functions is the integral-equation method. But this method fails to give the solution at any one of infinite number of discrete frequencies called irregular frequencies, which depend on the body geometry. Irregular frequencies are purely a consequence of the integral-equation method. Physically no such frequencies exist. Various modifications to the integral-equation methods have been proposed to overcome irregular frequencies. But such modified formulations invariably incur a significant increase in computational burden. They are apt to cause also degradation in the accuracy and stability of computations in the important low frequency range that usually lies below the fundamental (i.e. lowest) irregular frequency. It therefore makes little sense to apply any modified formulation indiscriminately. Fortunately, the irregular frequencies for conventional ships tend to lie beyond the frequency range of practical interest, that is, beyond the dominant excitation frequencies of ocean waves. But to restrict use of a modified formulation to where it is intended, we need to know what the irregular frequencies are for a particular hull form under consideration. For arbitrary-shaped bodies like conventional ships, this is not possible.

To overcome this difficulty, a simple analytical method has been developed to separate the frequency domain for calculations of ship motions and wave loads into two regimes. One is free of irregular frequencies and for which the well-established and well-validated ordinary integral equation method is appropriate. The other includes irregular frequencies and for which an alternative method for suppressing irregular frequencies is appropriate. The knowledge of the irregular-frequency-free regime makes it possible to incorporate a "switch" in the computer program for ship motions and waves loads to avoid the needless use of an alternative formulation in the all-important lower frequency regime. In addition, the present method can indicate if any anomalies in computational results are either genuine in the sense that they are owing to irregular frequencies or simply spurious as a result of a bug in the computer code.

Variational principles are used to derive the ordering of the magnitudes of the fundamental irregular frequencies according to shapes of the bodies. This method of estimating the lower bound to irregular frequencies for an arbitrary-shaped body is easier and more accurate than alternative methods that are currently available.

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Sommaire

Il est essentiel de pouvoir prédire le comportement d'un navire agité par la houle pour assurer sa sécurité structurale. Le travail d'élaboration d'un modèle analytique des mouvements d'un navire est surtout consacré au calcul des fonctions du potentiel de vitesse. La méthode la plus pratique de calcul des fonctions du potentiel de vitesse est la méthode des équations intégrales. Toutefois, cette méthode ne peut pas fournir une solution aux fréquences irrégulières — fréquences discrètes en nombre infini qui dépendent de la géométrie du corps. Ces fréquences irrégulières sont un sous-produit de la méthode des équations intégrales et n'ont aucune existence physique. Différentes modifications des méthodes des équations intégrales permettant de contourner le problème des fréquences irrégulières ont été proposées. Or, ces méthodes modifiées se traduisent invariablement par un accroissement important du temps de calcul. De plus, elles peuvent dégrader la précision et la stabilité des calculs dans l'intervalle important des basses fréquences, situé à des fréquences inférieures à la fréquence irrégulière fondamentale (la plus basse). Il ne serait donc guère logique d'utiliser sans distinction une formulation modifiée. Pour les navires classiques, heureusement, les fréquences irrégulières sont généralement plus élevées que l'intervalle des fréquences intéressantes du point de vue pratique, c'est-à-dire au-dessus des fréquences d'excitation dominantes de la houle océanique. Or, pour limiter à l'intervalle utile l'utilisation de la formulation modifiée, nous devons connaître quelles sont les fréquences irrégulières propres à la coque que nous étudions. Pour des corps aux formes arbitraires, comme les navires classiques, ceci n'est pas possible.

Pour contourner cette difficulté, nous avons mis au point une méthode analytique simple qui, aux fins du calcul des mouvements des navires et de la charge de la houle, divise le domaine des fréquences en deux régimes. Dans le premier, non contaminé par les fréquences irrégulières, l'on peut utiliser la méthode habituelle des équations intégrales, qui est bien établie et validée. Le second englobe les fréquences irrégulières et, pour les éliminer, il faut recourir à une autre méthode. Savoir quel régime n'est pas contaminé par les fréquences irrégulières a permis d'insérer, dans le logiciel de prévision des mouvements des navires et de la charge de houle, un « commutateur » qui évite d'utiliser inutilement une autre formulation dans le régime important des basses fréquences. Qui plus est, la méthode présentée ici peut révéler si des anomalies dans les résultats sont *normales* — elles seraient causées par les fréquences irrégulières — ou *fausses* — elles résulteraient d'une erreur de programmation.

Nous utilisons les principes variationnels pour établir l'ordre des amplitudes des fréquences irrégulières fondamentales en fonction des formes des corps. Cette méthode pour estimer la limite inférieure des fréquences irrégulières d'un corps aux formes arbitraires est plus simple et plus précise que les autres méthodes disponibles.

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Introduction

To solve the linearized boundary-value problems for oscillatory motions of a floating body (i.e. radiation and diffraction problems), the integral-equation method is widely used, because it reduces the original problem in an infinite exterior domain to one involving only the immersed surface of the body. But the integral-equation method fails to give solutions at irregular frequencies. By irregular frequencies, we mean those frequencies at which the inhomogeneous Fredholm integral equations of the 2nd kind for the radiation and diffraction potentials break down. Physically, no such frequencies exist. Irregular frequencies are strictly a consequence of the method. (For water-wave problems, see John [1], Ursell [2]; for a readable account of the problem concerning offshore platforms, see Murphy [3].) As a result, the transfer functions for, say, ship motions and wave loads, cannot be calculated at irregular frequencies. This makes it difficult to calculate the power spectra, the single most important parameter in the analysis of ship motions and wave loads in realistic confused seas. Locating the irregular frequencies is, therefore, not just of theoretical interest but also of considerable engineering importance.

In this note, a practical analytical method of estimating lower bounds to the fundamental (i.e. the lowest) irregular frequencies is presented for engineering applications. The method is based on the variational principles in the solution of the eigenvalue problem of wave-body interactions. While the proof of the principle is quite simple, I have not been able to find one that specifically addresses the boundary-value problem (BVP) of wave-ship interactions in the literature.

Irregular frequencies cannot normally be determined analytically, except for a small number of body shapes to which separation of variables can be effected, such as rectangular and circular shapes, and for which the irregular frequencies can be derived from a separation constant. Wehausen [4] mentions that there are only eleven body shapes for which the 3D Laplace equation can be separated and five for which the 2D one can be separated. For bodies of arbitrary shapes, John [1] obtains a lower bound to the wavenumbers $\{k\}$ corresponding to irregular frequencies: $k \le 1/H$, where $k = \omega^2/g$, ω is the angular frequency, and H is the draft of the body. Wu and Price [5] propose a method for estimating the irregular frequencies for an arbitrary 3D body using what they term the "equivalent box," a rectangular box having the same displacement volume as that of the arbitrary-shaped body of interest. Various empirical factors are used to determine the dimensions of the equivalent box. (Applicability of their method to 2D bodies is not shown in their paper.) The fact that the irregular frequencies so estimated can be either greater or smaller than the true values somewhat detracts from the usefulness of their method in practical problems. Sayer and Ursell [6] and Sayer [7] use the Rayleigh-Ritz principle to estimate the critical wavenumbers for a semi-circle and semi-ellipse. This method requires finding an appropriate trial function that satisfies the boundary conditions. This is by no means straightforward, even for separable domains like a circle or ellipse, and impossible for general shapes. As well, it can be shown (see, e.g. Rektorys [8]) that the sequence of approximate eigenvalues obtained by the Ritz method converges

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to the eigenvalues from above; that is, the true eigenvalues are always less than the values given by the Rayleigh quotient.

Some researchers have developed alternative integral-equation formulations to overcome the irregular frequencies and to obtain unique numerical solutions at all frequencies: *e.g.* Ohmatsu [9,10], Ogilvie and Shin [11], Martin [12], Lee and Sclavounos [13]. Valuable though these formulations are, it will be preferable to limit their use to the high frequency range that includes irregular frequencies for three reasons. First, because the alternative formulations all involve enlargement of boundary conditions and/or the integral equation, the computation burden is significantly increased, especially for 3D bodies. Second, depending on how they are implemented numerically, they are known to affect the accuracy and stability of computations in the lower frequency range that is free of irregular frequencies. Third, the irregular frequencies for conventional ship hull forms are generally high enough to be outside of the frequency range of practical interest.

As well, any hydrodynamicists who have worked on the computer programs for wave-body interactions would have encountered numerical instabilities in the outputs at varying frequencies. Determining whether such anomalous computational results are genuine in the sense that they are owing to irregular frequencies or spurious as a result of a bug in the computer code requires knowledge of a lower bound to the irregular frequencies.

It is useful, therefore, to establish a simple method of finding sharp lower bounds to irregular frequencies for ships and other floating bodies. This is the motivation for this work.

Analysis

Consider a ship executing simple harmonic motion in the free surface of water. We assume that water is inviscid and the flow irrotational. Deep water is assumed. The amplitudes of the motions of water and ships are to be small enough relative to some characteristic length of the ship to justify the linearization of the problem. Let S and A denote its mean wetted surface and the waterplane, respectively, and D the interior domain (figure 1). The Cartesian coordinates are such that A lies in $y=0$ and the y-axis points vertical upward. (The z-axis points out of paper.) Let the potential function for the radiated waves generated by the motion of the body in one of the six degrees of freedom of the motion be denoted by

$$\Phi_m(P,t) = \mathrm{Re}\phi_m(P)\dot{\eta}_m(t), \qquad m=1,2,...,6, \tag{1}$$

where P is a point in the fluid domain. Overdot indicates the time derivative. Denoting the amplitude of the ship's simple harmonic motion by η_{m0} and the angular frequency by ω, the complex oscillatory body displacement in the mth mode is expressed by

$$\eta_m(t) = \eta_{m0}e^{-i\omega t}, \qquad m=1,2,...,6 \tag{2}$$

For the steady oscillatory motions, the problem is thus reduced to solving for the time-independent complex potential ϕ_m. The potential ϕ_m is each determined as a solution of Laplace's equation,

$$\Delta\phi_m = 0, \quad m = 1, 2, ..., 6, \tag{3}$$

in the lower half-space ($y < 0$) outside the body, where Δ is the Laplace operator. In case where the *xyz*-axes, the equilibrium frame, move in the *z*-direction at constant speed U while maintaining the original orientation, ϕ_m is subject to the free-surface condition,

$$\left(i\omega + U\frac{\partial}{\partial z}\right)^2 \phi_m + g\frac{\partial \phi_m}{\partial y} = 0, \quad m = 1, 2, ..., 6 \tag{4}$$

on mean free surface $y = 0$ outside the body, where g is the gravitational acceleration. If the frequency is high and ship speed slow, then (4) can be approximated by

$$\frac{\partial \phi_m}{\partial y} - k\phi_m = 0 \ , \left(k = \frac{\omega^2}{g}\right), \quad m = 1, 2, ..., 6, \tag{5}$$

which is the case for $U = 0$. On the body, ϕ_m is required to satisfy the condition

$$\frac{\partial \phi_m}{\partial n} = \mathbf{V}_m \cdot \mathbf{n}, \quad m = 1, 2, ..., 6 \tag{6}$$

on mean wetted surface *S*. Here $\mathbf{V}_m$ is the prescribed velocity of the body in the *m*th mode and **n** is the unit vector normal to *S* pointing into the body. For the velocity potential for the diffracted wave by a ship, say ϕ_7, the right-hand side of (6) is replaced by $\partial\phi_I/\partial n$, where ϕ_I is the velocity potential for a uniform plane progressive incident wave of angular frequency ω. In addition, ϕ_m, $m = 1, 2, ..., 7$, satisfies the Summerfeld radiation condition at infinity, and the Neumann condition $\partial\phi_m/\partial y = 0$ on the sea bottom.

The main problem in the analysis of wave-ship interactions consists of calculating added mass and damping. Added mass accounts for the effects produced by the water that is accelerated by the oscillatory motion of the ship, and damping for the energy carried away by the waves generated by the motions of the ship. The added-mass coefficient A_{lm} is the ratio of hydrodynamic force in the *l*-mode in phase with body acceleration in the *m*-mode, and the damping coefficient B_{lm} is the ratio of hydrodynamic force in the *l*-mode in phase with body velocity in the *m*-mode. In linear theory, these ratios are independent of the magnitudes of acceleration and velocity, respectively. By taking the cross-coupling among different modes of motions into account, the generalized hydrodynamic force (or moment) F_{lm} in the *l*th direction due to the oscillation of the body in the *m*th direction can be expressed as

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$$F_{lm} = A_{lm}\ddot{\eta}_m + B_{lm}\dot{\eta}_m, \quad l,m = 1,2,...,6. \tag{7}$$

The force (or moment) in the lth mode, F_l, is then obtained by

$$F_l(t) = \sum_{m=1}^{6} F_{lm} = \sum_{m=1}^{6}\left(A_{lm}\ddot{\eta}_m + B_{lm}\dot{\eta}_m\right), \quad l = 1,2,...,6. \tag{8}$$

Added-mass and damping coefficients can be expressed in terms of the velocity potential ϕ_m as follows:

$$A_{lm} + \left(\frac{i}{\omega}\right)B_{lm} = \rho\iint_S n_l(q)\phi_m(q)dS_q, \quad l,m = 1,2,...,6. \tag{9}$$

In the integral-equation method for solving the boundary-value problem (3)–(6) plus the radiation condition and the sea-bed condition, the potential ϕ_m is usually expressed as a source distribution σ_m on the wetted surface:

$$\phi_m(p) = \iint_S \sigma_m(q)G(p,q)dS_q + \frac{U^2}{g}\int_\Gamma \sigma_m(q)G(p,q)n_3(q)d\Gamma_q, \quad p,q \in S, \quad m = 1,2,...,6 \tag{10}$$

where p and q denote the field and source points, respectively, on the wetted surface, Γ is the contour at the intersection of the hull and the calm-water waterline, $G(p,q)$ is the Green's function, and n_3 is the z-component (i.e. along the longitudinal axis) of $\mathbf{n}$.

The Green's function satisfies Laplace's equations in the lower half-space outside the body except at $p = q$, the free-surface condition (4) or (5), the sea-bottom condition, and the radiation condition at infinity. The Green's function for the 3D problem in deep water for a ship advancing with speed U where the free-surface condition (4) is applicable expressed by [14]

$$\begin{aligned} 4\pi G(x,y,z;\xi,\eta,\zeta) = {} & \frac{1}{r} + \frac{1}{r_1} + \frac{2g}{\pi}\int_0^{\gamma} d\theta \int_0^{\infty} F(\theta,\beta)d\beta + \frac{2g}{\pi}\int_{\gamma}^{\pi/2} d\theta \int_{L_1} F(\theta,\beta)d\beta \\ & + \frac{2g}{\pi}\int_{\pi/2}^{\pi} d\theta \int_{L_2} F(\theta,\beta)d\beta \end{aligned} \tag{11}$$

where

$$\begin{aligned} r^2 &= (x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2, \\ r_1^2 &= (x-\xi)^2 + (y+\eta)^2 + (z-\zeta)^2, \end{aligned} \tag{12}$$

$$F(\theta,\beta) = \frac{\beta e^{\beta[(y+\eta)+i(x-\xi)\cos\theta]}\cos\left[\beta(z-\zeta)\sin\theta\right]}{g\beta - (\omega + \beta U\cos\theta)^2}, \tag{13}$$

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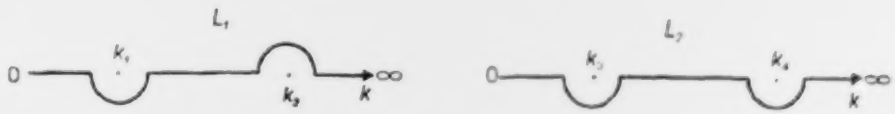
$$\gamma = \begin{cases} 0 & \text{if } \tau < \frac{1}{4}, \\ \arccos\left(\frac{1}{4\tau}\right) & \text{if } \tau \geq \frac{1}{4}. \end{cases} \tag{14}$$

$$\tau = \frac{U\omega}{g}. \tag{15}$$

The contours of integration L_1 and L_2 in (11) are defined by



where the singularities are located at

$$\begin{aligned} \sqrt{gk_1} &= \sqrt{gk_3} = \frac{1-\sqrt{1-4\tau\cos\theta}}{2\tau\cos\theta}\omega \\ \sqrt{gk_2} &= -\sqrt{gk_4} = \frac{1+\sqrt{1-4\tau\cos\theta}}{2\tau\cos\theta}\omega \end{aligned} \tag{16}$$

For a zero-forward speed, the line integral term of the RHS of (10) vanishes. As well, for a slender-ship for which $|n_3|$ is sufficiently small, the line integral term is small compared with the first term, so it can be neglected. In either case, (10) is reduced to

$$\phi_m(p) = \iint_S \sigma_m(q) G(p,q)\, dS_q, \quad p,q \in S, \ m = 1,2,\ldots,6, \tag{17}$$

The Green's function that does not depend on speed explicitly can be written as [14]

$$4\pi G(x,y,z;\xi,\eta,\zeta) = \frac{1}{r} + \mathrm{PV}\int_0^\infty \frac{\beta+k}{\beta-k} e^{\beta(y+\eta)} J_0(\beta R)\, d\beta + i2\pi k e^{k(y+\eta)} J_0(kR) \tag{18}$$

where (x,y,z) and (ξ,η,ζ) are coordinates of p and q, respectively, J_0 is the Bessel function of the first kind of zero order, and

$$\begin{aligned} r^2 &= R^2 + (y-\eta)^2, \\ R^2 &= (x-\xi)^2 + (z-\zeta)^2. \end{aligned} \tag{19}$$

The Green's function G of the form (18) is sometimes called the zero-forward speed Green's function. But it does account for the effect of ship speed on the increase in the oscillatory motion

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when ω is equal to the encounter frequency. In practice, (18) is often used in place of (11), even for a non-zero forward speed case. This is because the numerical evaluation of the latter and its derivatives is extremely complex, and gives rise to significant instability and errors in the results. This note addresses the case where the free-surface condition is expressed by (5); hence the Green's function is given by (18).

For the 2D problem in deep water, the Green's function can be expressed as

$$G(z;\zeta) = \frac{1}{2\pi}\log\left(\frac{R}{R_1}\right) + \frac{1}{\pi}\mathrm{PV}\int_0^\infty \frac{e^{\beta(y+\eta)}}{k-\beta}\cos\beta(x-\xi)d\beta - ie^{\kappa(y+\eta)}\cos k(x-\xi). \tag{20}$$

Note that in the 2D Green's function (20), z and ζ are complex numbers representing the coordinates of p and q, respectively:

$$\begin{aligned} z &= x + iy, \\ \zeta &= \xi + i\eta, \end{aligned} \tag{21}$$

and r and r_1 in (20) are defined as

$$\begin{aligned} R^2 &= (x-\xi)^2 + (y-\eta)^2, \\ R_1^2 &= (x-\xi)^2 + (y+\eta)^2. \end{aligned} \tag{22}$$

Since Green's function G satisfies Laplace's equations and the boundary conditions on the free surface and on the sea bottom, and the radiation condition at infinity, ϕ_m as expressed as (10) or (17) needs to satisfy only the body boundary condition (6).

Substituting (17) into the boundary condition (6) gives a Fredholm integral equation of the second kind for the source distribution σ_m for the mth mode of motion:

$$-\frac{1}{2}\sigma_m(p) + \int_S \sigma_m(q)\frac{\partial G(p,q)}{\partial n_p}dS_q = \mathbf{V}_m \cdot \mathbf{n}, \quad p,q \in S,\ m = 1,2,\ldots,6, \tag{23}$$

Now, from Green's formula,

$$\int_S \left\{\phi(q)\frac{\partial G(p,q)}{\partial n_q} - G(p,q)\frac{\partial \phi(q)}{\partial n_q}\right\}dS_q = -\frac{1}{2}\phi(p), \quad p,q \in S, \tag{24}$$

$$\frac{\partial}{\partial n_p}\int_S \left\{\phi(q)\frac{\partial G(p,q)}{\partial n_q} - G(p,q)\frac{\partial \phi(q)}{\partial n_q}\right\}dS_q = -\frac{1}{2}\frac{\partial \phi(p)}{\partial n_p}, \quad p,q \in S. \tag{25}$$

Thus if $\phi = 0$ on S, we have from (24),

$$\int_S G(p,q)\frac{\partial\phi(q)}{\partial n_q}dS_q = 0, \quad p,q\in S, \tag{26}$$

and from (25),

$$\int_S \frac{\partial G(p,q)}{\partial n_p}\frac{\partial\phi(q)}{\partial n_q}dS_q = \frac{1}{2}\frac{\partial\phi(p)}{\partial n_p}, \quad p,q\in S. \tag{27}$$

Following John [1], we introduce integral operators $\mathcal{K}_0$ and $\mathcal{M}_0$:

$$\mathcal{K}_0\nu = \int_S G(p,q)\nu(q)dS_q, \quad p,q\in S, \tag{28}$$

$$\mathcal{M}_0\nu = \int_S \frac{\partial G(p,q)}{\partial n_p}\nu(q)dS_q, \quad p,q\in S. \tag{29}$$

Then (26) and (27) can be expressed in the forms:

$$\mathcal{K}_0\nu = 0, \tag{30}$$

$$\left(\frac{1}{2}\mathcal{I} - \mathcal{M}_0\right)\nu = 0, \tag{31}$$

where $\mathcal{I}$ is the identity operator. John [1] has established that (30) and (31) are equivalent and that (31) is satisfied by the value of the normal derivative $\nu = \partial\phi/\partial n$ of the solution ϕ of the following interior BVP:

$$\begin{aligned} \Delta\phi &= 0 \quad \text{in } D \\ \frac{\partial\phi}{\partial y} - k\phi &= 0 \quad \text{on } A \quad \left(k = \omega^2/g\right) \\ \phi &= 0 \quad \text{on } S \end{aligned} \tag{32}$$

where ϕ is expressed by (17) and represents any of ϕ_m, $m = 1,2,...,7$. (Incidentally, changing the Dirichlet condition on S to the Neumann condition yields the sloshing problem, or waves in basins of bounded extent: see, *e.g.* Wehausen and Laitone [14].) The BVP (32) will only have the trivial solution, $\phi = 0$, but at certain values of wavenumber k that make 2 an eigenvalue of the operator $\mathcal{M}_0$, nontrivial solutions exist. John [1] called the frequencies corresponding to such critical wavenumbers the irregular frequencies for the radiation and diffraction problems. It can be seen from the definition of the integral operator $\mathcal{M}_0$, that the irregular frequencies are strictly a consequence of the kernel of the integral operator, or specifically, of Green's function (18) for 3D problems and (20) for 2D problems.

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It is not generally possible to solve the BVP (32) analytically, except for a family of boundary shapes for which Laplace's equation is separable. For the radiation and diffraction problems involving an arbitrary-shaped floating body, therefore, solving the eigenvalue problem (32) for the irregular frequencies is just as difficult as solving the original problem.

One way of circumventing the necessity of having to solve the BVP (32) directly in order to determine at least the all-important fundamental eigenvalue k_1 for the given arbitrary-shaped body is to look for a kind of comparison theorem; that is, to show that the eigenvalues associated with the BVP (32) satisfy the domain monotonicity: if domain D contains domain D', then ith eigenvalue k_i for D is smaller than the nth eigenvalue k_i of D' for each i. Then it is possible to obtain a lower bound for k_1 for an arbitrary-shaped body like a conventional ship by simply circumscribing the body with one of the families of geometries for which Laplace's equation is separable and the eigenvalues can calculated explicitly. The closer the circumscribing body is to the arbitrary-shaped body in question, the closer the estimated k_1 will be to the actual value for the arbitrary-shaped body. This is the motivation for this study. Note that the BVP (32) is not the usual type of eigenvalue problem because the eigenvalue k does not appear in the differential equation but in the boundary conditions. (For the domain monotonicity for the usual eigenvalue problem for sound waves involving the Helmholtz equation or the Schrödinger equation subject to homogeneous boundary conditions, see, *e.g.*, Courant and Hilbert [15], Morse and Feshbach [16].)

For our problem, we recast BVP (32) as

$$\mathcal{N}(\phi) = k\mathcal{I}(\phi), \tag{33}$$

where $\mathcal{N}$ represents the normal-derivative operator (the normal vector points out of D) and $\mathcal{I}$ the identity operator, both defined on a Hilbert space $L_2(A)$ of square-integrable functions on A. The admissible functions ϕ are continuous in $\overline{D}$ with piecewise continuous first and second derivatives in D, are harmonic in D, and vanish on S. By the Rayleigh-Ritz principle, the lowest eigenvalue k_1 of the operator $\mathcal{N}$ is bounded above by the stationary value of the quotient $\iint_A \phi\mathcal{N}(\phi)dS / \iint_A \phi\mathcal{I}(\phi)dS$. Now, in the problem at hand,

$$\frac{\iint_A \phi\mathcal{N}(\phi)dS}{\iint_A \phi\mathcal{I}(\phi)dS} = \frac{\iint_A \phi\frac{\partial\phi}{\partial n}dS}{\iint_A \phi^2 dS} = \frac{\iint_{A\cup S} \phi\frac{\partial\phi}{\partial n}dS}{\iint_A \phi^2 dS} = \frac{\iiint_D (\nabla\phi\cdot\nabla\phi)dV}{\iint_A \phi^2 dS}, \tag{34}$$

where we used Green's first identity and the fact that ϕ is harmonic in D and vanishes on S. Since $\mathcal{N}$ is a positive definite operator, its eigenvalues $\{k_i\}$ are real and positive, and form a countably infinite set. Hence the spectrum $\{k_i \mid i = 1,2,...\}$ of the operator $\mathcal{N}$ has a greatest lower bound (glb), if not a minimum. The eigenfunctions $\{\phi_i\}$ corresponding to $\{k_i\}$ are complete in A (but not in D) and orthogonal in A. Since k_1 is the absolute minimum of (34),

$$k_1 = \text{glb}\left\{\frac{\iiint_D (\nabla\phi \cdot \nabla\phi) dV}{\iint_A \phi^2 dS}\right\}. \tag{35}$$

This agrees with the result given by John [1]. The variational principle for $\{k\}$ just established enables us to arrive at the domain monotonicity for our problem as follows. First, consider the case where the domain D and a subdomain D', which is wholly inside of D, have the same free surface $A = A'$ and let E be the domain between D and D', i.e., $E = D \setminus D'$ (figure 1). Let K denote the class of the admissible functions ϕ defined on D, and K' the class of the admissible functions ϕ' defined on D'. Let K^* be another class of admissible functions ϕ^* defined on D such that $\phi^*(p) = \phi'(p)$ for $p \in D'$, and ϕ^* and $\nabla\phi^*$ vanish on $\overline{E}$, the closure of E (i.e. $\overline{E}$ is a set of points of equilibrium for ϕ^*). So $K^* \subset K$. Thus if k_1, k_1', and k_1^* are respectively the lowest eigenvalues with respect to K, K', and K^*, then k_1^* cannot be less than k_1 because k_1 is the absolute minimum among the functions in K. Thus $k_1 \le k_1^*$. But by definition, the restriction of K^* to D' coincides with K'. So $k_1^* = k_1'$, and we obtain the inequality:

$$k_1 \le k_1'. \tag{36}$$

In the case where the free surface A' of D' lies entirely in A, the size of K^* is further reduced compared with that of K because of the imposition of a stronger boundary condition $\phi^* = 0$ on parts of A, and so (36) holds *a fortiori*. This is one of the consequences of the principles of the calculus of variations (see, e.g. [15], p. 407). (Note that the sense of such monotonicity of the eigenvalues as (36) depends on the boundary conditions; that is, the inequality (36) is reversed in the case of the sloshing problem with the identically defined domains D and D'; see, e.g. Troesch [17]). The interior BVP (32) derived by John [1] corresponds to the diffraction problem as well as to the six degrees of motion in the radiation problem, so $m = 1, 2, \ldots, 7$ in the BVP (1)-(6). Thus the set K of the eigenvalues associated with the BVP (32) will include the irregular frequencies for all modes.

In making use of the monotonicity (36) of the eigenvalues associated with the BVP (32), the most desirable candidate for the circumscribing body from the point of view of accuracy is that which encloses the given floating body as closely as possible. But the eigenvalues of the circumscribing body chosen based on this criterion are not always amenable to explicit calculation, like the eigenvalues for a body whose cross section is a uniform semi-circle. The most convenient shape of the circumscribing body is then a rectangular prism, for which the separation of variables can be most easily effected and the eigenvalues can be expressed in closed form. Let the dimensions of the enclosing prism be $L \times B \times H$. Then applying the method of separation of variables to the interior problem (32) yields the eigenvalues

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$$k_{ij} = \kappa_{ij} \coth\{\kappa_{ij} H\}, \tag{37}$$

where

$$\kappa_{ij} = \pi\sqrt{(i/L)^2 + (j/B)^2}, \quad i, j = 1, 2, \ldots, \tag{38}$$

for a 3D rectangular prism, and

$$k_j = (j\pi/B)\coth(j\pi H/B), \quad j = 1, 2, \ldots \tag{39}$$

for a 2D rectangular section. By way of examples, contour plots of the eigenfunctions corresponding to the two lowest eigenvalues or irregular wavenumbers, k_1 and k_2, for a rectangular prism are shown in figure B1 in Annex B. Combining these and (36), we obtain the following lower bounds for k_1 for arbitrary-shaped bodies:

$$k_1 \geq \kappa_1 \coth(\kappa_1 H), \tag{40}$$

where

$$\kappa_1 = \begin{cases} \pi\sqrt{L^{-2} + B^{-2}} & \text{for 3D bodies,} \\ \pi/B & \text{for 2D bodies.} \end{cases} \tag{41}$$

Obviously, the greater the block coefficient of the ship, the closer the estimates of k_1 by (40) will be to the true values. By comparison, the lower bound given by John [1] is

$$k_1 \geq 1/H. \tag{42}$$

Since $\coth x \to 1/x$ as $x \to 0$, (42) can be regarded as the limiting case of (40) when $\kappa_1 H \to 0$.

For almost all conventional ships,

$$10 < \frac{L}{H} < 30, \; 1.8 < \frac{B}{H} < 5.0, \text{ and } 3.5 < \frac{L}{B} < 10, \tag{43}$$

approximately. For these ranges of the ratios, the fundamental irregular frequency,

$$\omega_1 = \sqrt{g k_1}, \tag{44}$$

is plotted against the draft H in figure 2. The frequency regime below a line in figure 2 is entirely free of irregular frequencies and so the well-established and well-validated codes based on the ordinary integral-equation method gives a unique solution and should be used. The frequency regime above a line contains irregular frequencies, and so it is only for this frequency regime that an alternative formulation for suppressing irregular frequencies needs to be used. The difference

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between the solid-line curves and the broken line in figure 2 represents the improved accuracy of the present method over John's method.

For small B/L ratios, the expression of κ_1 for 3D bodies can be expressed by the Taylor series as

$$\kappa_1 = \frac{\pi}{B}\sqrt{1+\left(\frac{B}{L}\right)^2} \approx \frac{\pi}{B}\left[1+\frac{1}{2}\left(\frac{B}{L}\right)^2\right] \tag{45}$$

For the range of L/B for conventional ships given in (43), (45) yields

$$1.005\frac{\pi}{B} \le \kappa_1 \le 1.041\frac{\pi}{B}.$$

Thus, the irregular frequencies for conventional ships may be considered to be independent of the ship length and depend only on the value of the H/B ratio of the enclosing prism.

Examples

3D calculations

A frigate

The body plan for the hull form below the design waterline is shown in figure 3 (top). The dimensions of the hull are $L \times B \times H$ = 122.85 m x 14.04 m x 4.68 m, so $L/H = 26.3$, $B/H = 3.00$, and $L/B = 8.75$. The ω_1-versus-H line corresponding to these ratios is drawn in figure 2. The added masses and damping for zero forward speed were calculated using a 3D panel-method code. The results for heave are shown in figure 3 (bottom), and those for sway and roll in figure 4. The spikes in these and the rest of figures of added masses and damping give forms to the numerical instabilities. According to John [1], $\omega_1 \ge$ 1.45 rad/s ($\omega_1/\sqrt{g/L} \ge 5.123$, $k_1 \ge 1/4.68 = 0.214$ m^{-1}). By the present method, (40) gives $k_1 \ge 0.288$ m^{-1}or $\omega_1 \ge 1.68$ rad/s ($\omega_1/\sqrt{g/L} \ge 5.94$). For the method of Wu and Price [5], the areas of the waterplane (A_w=1293.6 m^2), the midsection (A_m = 53.16 m^2), and the profile of the centreplane ("central longitudinal section" [5]) (A_c = 507.5 m^2) must be determined numerically. Substituting these values into equations (7)–(9) of Wu and Price [5] (see equations (A.1) –(A.3) in Annex A) gives ω_1 = 1.92 rad/s ($\omega_1/\sqrt{g/L}$ = 6.81, k_1 = 0.378 m^{-1}) for heave and ω_1 = 2.26 rad/s ($\omega_1/\sqrt{g/L}$ = 7.98, k_1 = 0.518 m^{-1}) for sway and roll (see Annex A for different values of ω_1 for heave and for sway and roll).

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In figures 3 and 4, the lower bounds for the fundamental irregular frequency predicted by John and the present methods are indicated by the arrows A and B, respectively the fundamental irregular frequency estimated by Wu and Price is indicated by C. For the reason mentioned in the Analysis section above, the fundamental irregular frequencies for sway and roll (asymmetrical motions) will be no smaller than that for heave (symmetrical motion). For this particular hull form, the instabilities in the calculated added mass and damping indicate that the fundamental irregular frequencies for sway and roll are about 25 percent larger than that for heave.

Figures 3 and 4 provide an important fact: the computations of added masses and damping are apt to be unstable for frequencies above the fundamental irregular frequency. So the alternative formulations mentioned in the previous section should be used for calculation of added masses and damping at all frequencies above the fundamental irregular frequency. (The locations of 2nd, 3rd, and other higher irregular frequencies are, therefore, of little practical importance.)

Note that the block coefficient (C_B) is of the hull is 0.51; in other words, the submerged portion of the hull is only 51 percent of the enclosing rectangular body. Significantly, even for such a fine hull form, figure 3 (bottom) shows that the present method gives a fairly sharp prediction of the lower bound.

A container ship

The hull form below the design waterline is shown in figure 5 (top). The dimensions of the hull are $L \times B \times H =$ 160.0 m x 24.65 m x 9.3 m, so $L/H = 17.2$, $B/H = 2.65$, and $L/B = 6.49$. Added masses and damping for zero forward speed were calculated using the same 3D panel-method code as used for the frigate above. The results for heave are shown in figure 5 (bottom), and those for sway and roll in figure 6. John's method [1] gives $\omega_1 \geq$ 1.03 rad/s ($\omega_1/\sqrt{g/L} \geq$ 4.15, $k_1 \geq$ 1/4.68 = 0.108 m^{-1}), and the present method, (40), gives $k_1 \geq$ 0.155 m^{-1} or $\omega_1 \geq$ 1.23 rad/s ($\omega_1/\sqrt{g/L} \geq$ 4.98). For the method of Wu and Price [5], $A_\infty = 2485.5$ m^2, $A_m = 225.0$ m^2, and $A_c = 1352$ m^2 are calculated and substituted into equations (A.1) –(A.3) in Annex A, resulting in ω_1 = 1.31 rad/s ($\omega_1/\sqrt{g/L} = 5.28$, $k_1 = 0.174$ m^{-1}) for heave and ω_1 = 1.63 rad/s ($\omega_1/\sqrt{g/L} =$ 6.57, $k_1 = 0.270$ m^{-1}) for sway and roll. In figures 5 and 6, the lower bounds for the fundamental irregular frequency predicted by John and the present methods are indicated by the arrows A and B, respectively, and the fundamental irregular frequencies estimated by Wu and Price is indicated by C. For this hull form, the fundamental irregular frequencies for sway and roll (asymmetrical motions) appear to be about 28 percent larger than that for heave (symmetrical motion).

In figures 5 and 6, the computations become increasingly unstable for higher frequencies above the fundamental irregular frequency, as noted in the case of the frigate. For this container ship, C_B = 0.562, which is about 10 percent greater than that for the frigate. As a result, the predicted fundamental irregular frequency for heave predicted for this hull by the present method in figure 5 (bottom) is sharper than that for the frigate.

2D calculations

a) An amidships section

Examples of the 2D heave added mass and damping per unit length of the hull for an amidships section (B = 12.8 m, H = 4.0 m) of figure 7 (top) computed by a 2D source-distribution code are shown in figures 7 (bottom) and 8. The arrows A and B indicate the lower bounds of the fundamental irregular frequency according to John and the present method, respectively. The method of Wu and Price [5] cannot be applied to 2D bodies. The present method predicts the lower bound of the fundamental irregular frequency accurately in figure 7. From the irregular spikes in figures 7 and 8, the fundamental irregular frequencies for asymmetric motions (sway and roll) are seen to be about 18 percent greater than that for the symmetric motion (heave) for this particular hull form.

b) A surface-piercing circular section

For the surface piercing section (B = 12.0 m, H = 11.0 m) of figure 9 (top), added masses and damping per unit length for heave and sway calculated by the same 2D source-distribution code used above are shown in figures 9 (bottom) and 10 (top), and added moment and damping for roll in figure 10. For this section, the present method, not to mention John's method, noticeably under-predicts the fundamental irregular frequency for heave (by about 18 percent). This may be attributed to a small beam-to-draft ratio of this section: B/H = 1.09. This value is outside of the range for conventional ships, (43), and is about one-third of the value for the amidship section of figure 7. For this section, the computed added mass and damping for sway in figure 10 (top) show no noticeable effect of irregular frequencies over the frequency range shown. The magnitudes of the added moment and damping for rolling motion are infinitesimally small: see figure 10 (bottom). So the irregularities seen in the figure 10 are practically negligible.

c) Floating semi-circle

According to John's result (42), for a submerged semi-circle of radius $a\,(=H)$, $k_1H \geq 1$. Setting $B \times H = 2a \times a$, the present method, (40), gives a lower bound $k_1H \geq 1.713$. This value is only 6 percent smaller than the true value for k_1H, which lies between 1.818 and 1.819 (Ohmatsu [9]); see figure 11. (Note: the wavenumber is denoted by K in [9].) The upper bound for k_1H predicted by the method of Sayer and Ursell [7] is 1.822. Figure 11 indicates that the former is more than 30 percent greater than the latter for this particular sectional form.

Concluding remarks

In his classical work, John [1] derives a lower bound of the irregular frequencies for an arbitrary-shaped floating body. His result makes use of only the information about the draft of the body. The present method makes use of the additional information about the horizontal dimensions of the submerged part of the body as well. As a result, sharper estimates of lower bounds of irregular frequencies than John's are obtained. The method is useful in determining the frequency range that is free of irregular frequencies fairly accurately and easily for arbitrary-shaped floating bodies such as conventional ships.

By the present method, an alternative integral-equation formulation for suppressing irregular frequencies need not be used indiscriminately but only when it is needed, which is usually rare for conventional hull forms. Because such alternative formulations are purchased at the price of increased computational burden, the present method ensures efficient computation of ship motions and wave loads.

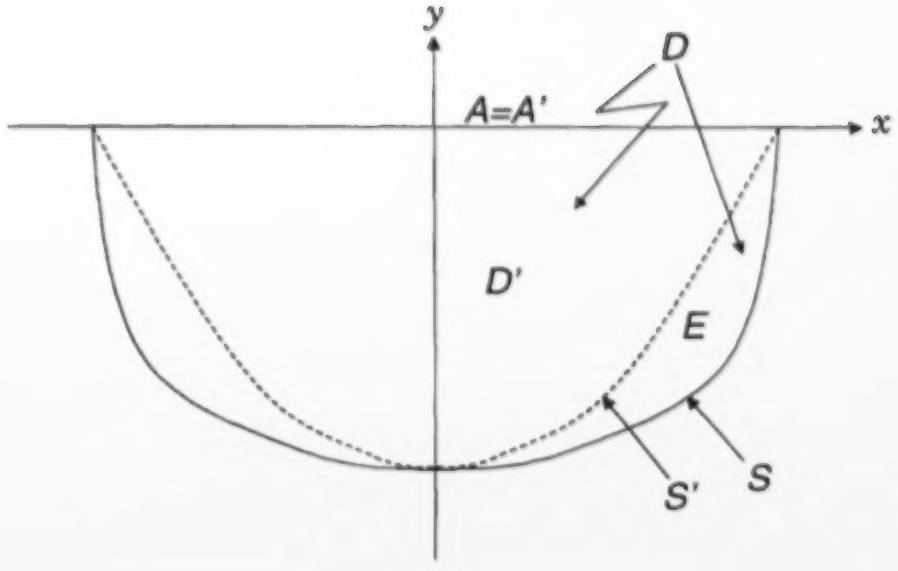

Figure 1. *Domain D and subdomain D', whose mean wetted surfaces are S and S', and calm-water surfaces A and A', respectively. Here D' is completely included in D. The domain between D and D' is denoted by E, that is, E = D\D', in the main text.*

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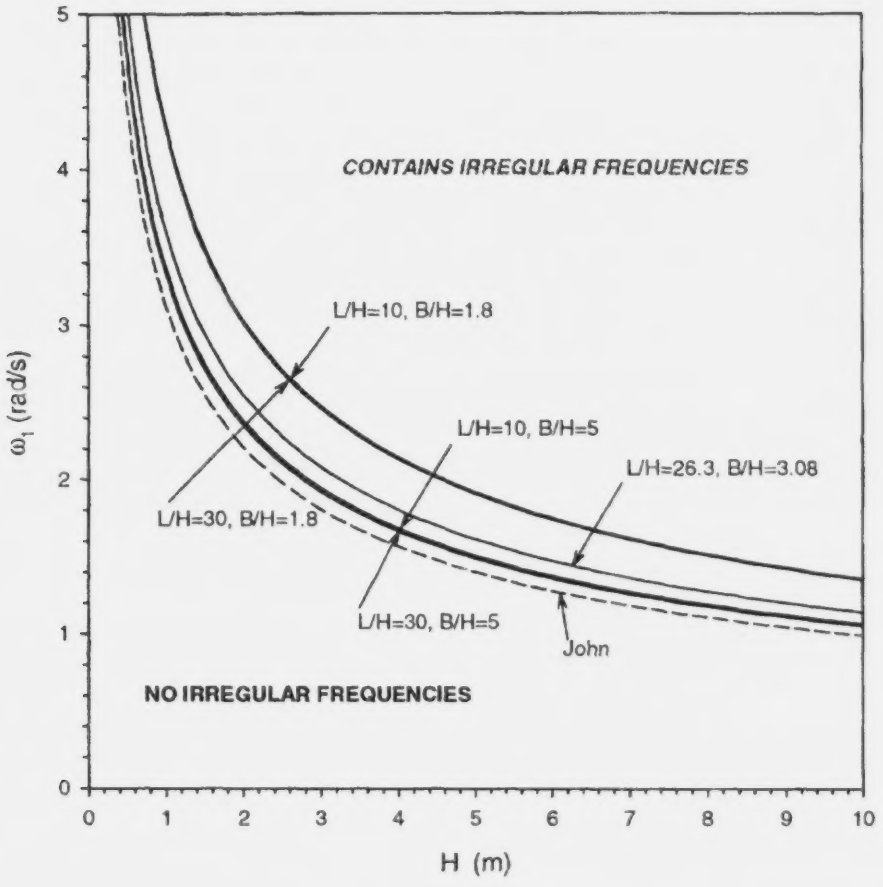

Figure 2. *Fundamental irregular frequencies for 3D floating bodies as a function of H for various values of L/H and B/H predicted by the present method (———) and John (- - - -). A line separates the frequency domain into two regimes: one below is free of irregular frequencies, and that which is above a line contains irregular frequencies. The line for L/H = 26.3, B/H = 3.08 corresponds to the frigate hull of figures 3 and 4.*

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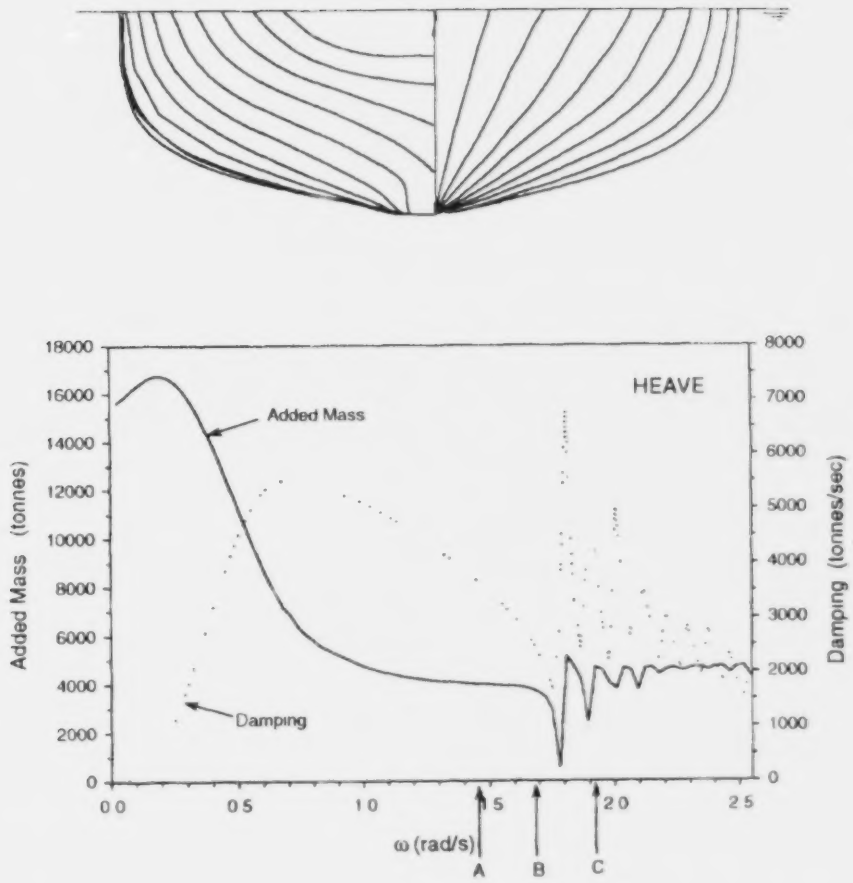

***Figure 3**. Body plan below WL of a frigate (top) and added masses and damping for heave computed by 3D panel-method for a frigate at zero forward speed (bottom). L x B x H = 122.85 m x 14.04 m x 4.68 m. The arrows A and B indicate the lower bounds of irregular frequencies predicted by John [1] and the present method, respectively, and C indicates the 1st irregular frequency predicted by Wu and Price [5].*

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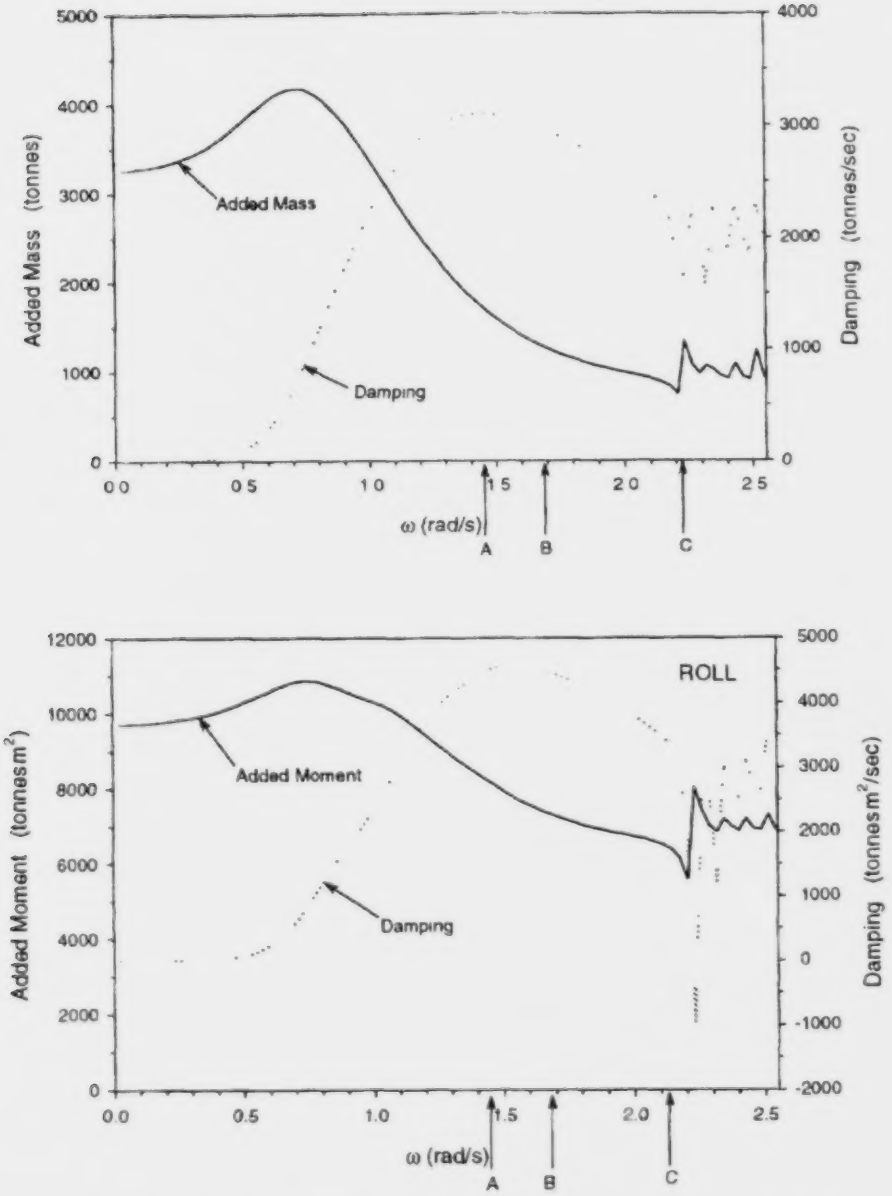

Figure 4. Added mass and damping for sway (top) and added moment and damping for roll (bottom) computed by 3D panel-method for the frigate at zero forward speed. The arrows A and B indicate the lower bounds of irregular frequencies predicted by John [1] and the present method, respectively, and C indicates the 1st irregular frequency predicted by Wu and Price [5].

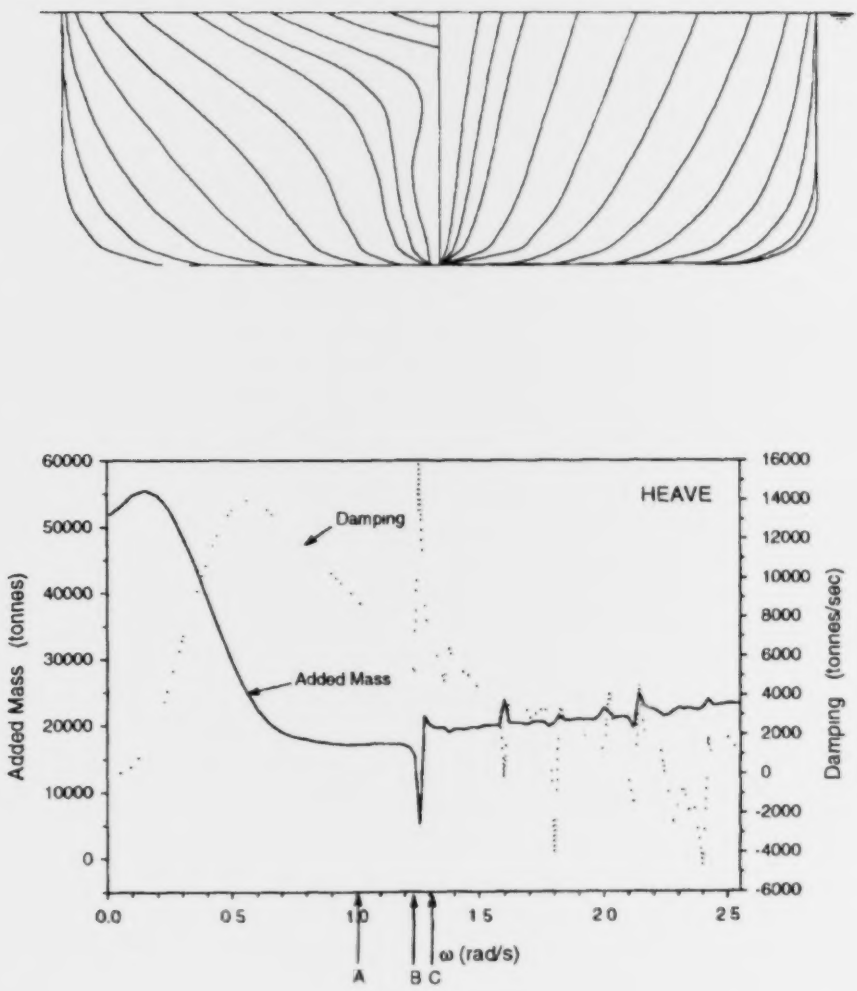

Figure 5. *Body plane below WL of a container ship (top) and added masses and damping for heave computed by 3D panel-method for a container ship at zero forward speed (bottom). L x B x H = 160 m x 24.65 m x 9.3 m. The arrows A and B indicate the lower bounds of irregular frequencies predicted by John [1] and the present method, respectively, and C indicates the 1st irregular frequency predicted by Wu and Price [5].*

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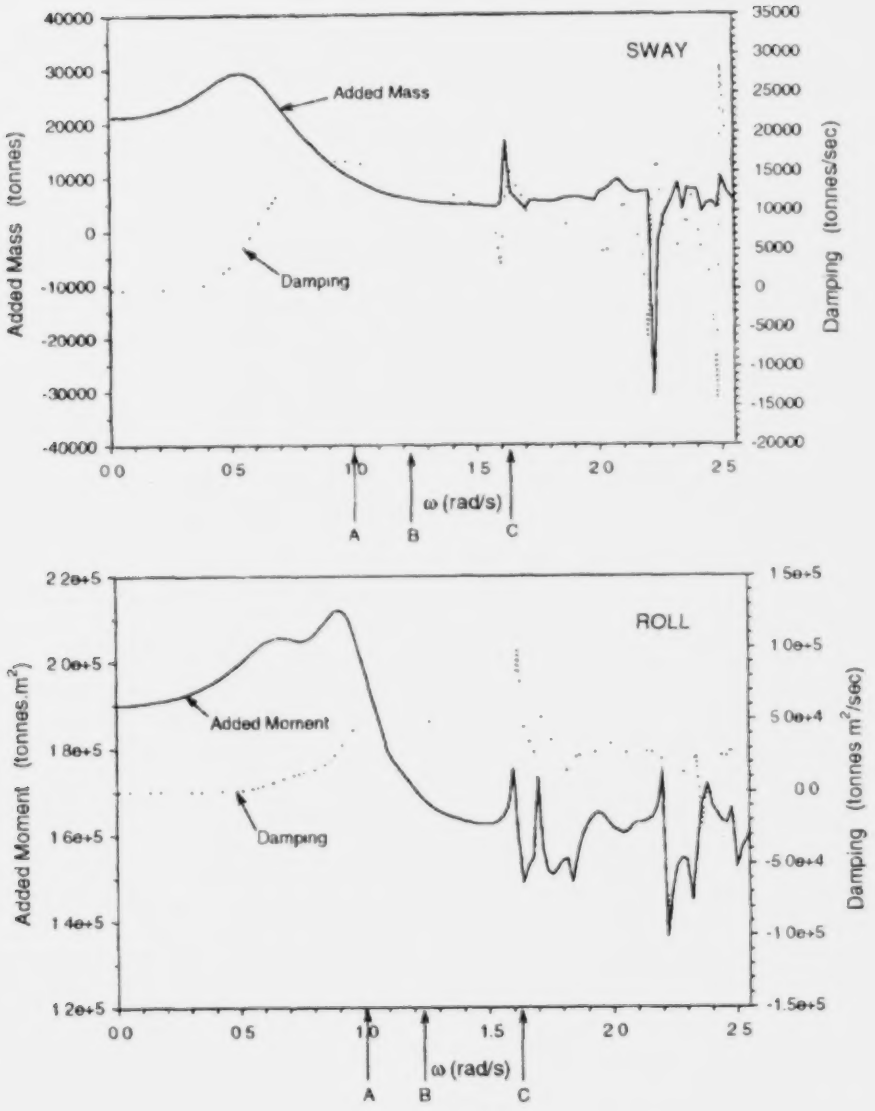

Figure 6. *Added mass and damping for sway (top) and added moment and damping for roll (bottom) computed by 3D panel-method for the container ship at zero forward speed. The arrows A and B indicate the lower bounds of irregular frequencies predicted by John [1] and the present method, respectively, and C indicates the 1st irregular frequency predicted by Wu and Price [5].*

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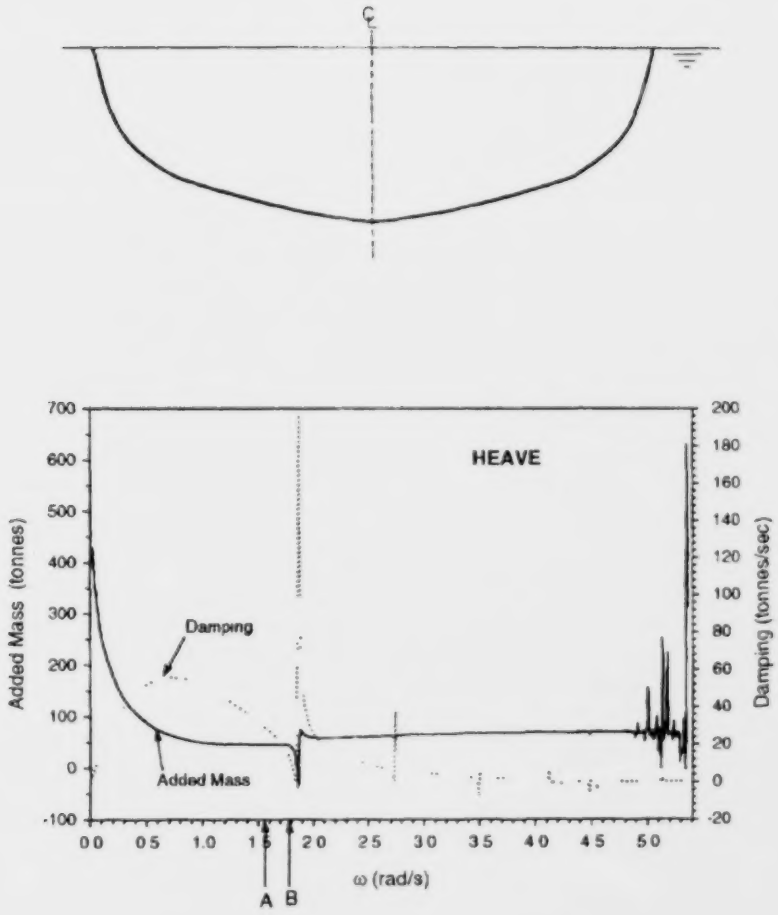

Figure 7. *Sectional contour of an amidship section (top) and heave added mass and damping (bottom) for the section computed by a 2D source-distribution method.). B x H = 12.8 m x 4.0 m. The arrows A (ω = 1.566 rad/s) and B (ω =1.787 rad/s) indicate lower bounds of irregular frequencies predicted by John [1] and the present method, respectively.*

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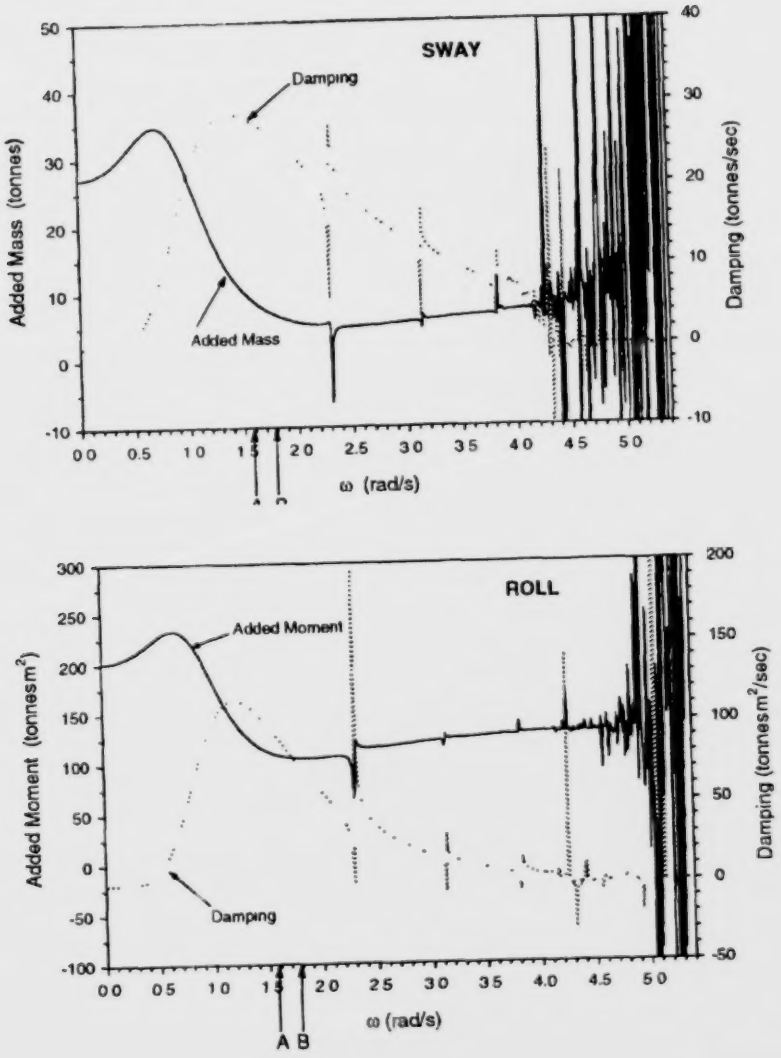

Figure 8. *Sway added mass and damping (top) and roll added moment and damping (bottom) for the amidship section of figure 7 computed by a 2D source-distribution method. The lower bounds of irregular frequencies predicted by John [1] and the present method are indicated by A (ω = 1.566 rad/s) and B (ω = 1.787 rad/s), respectively.*

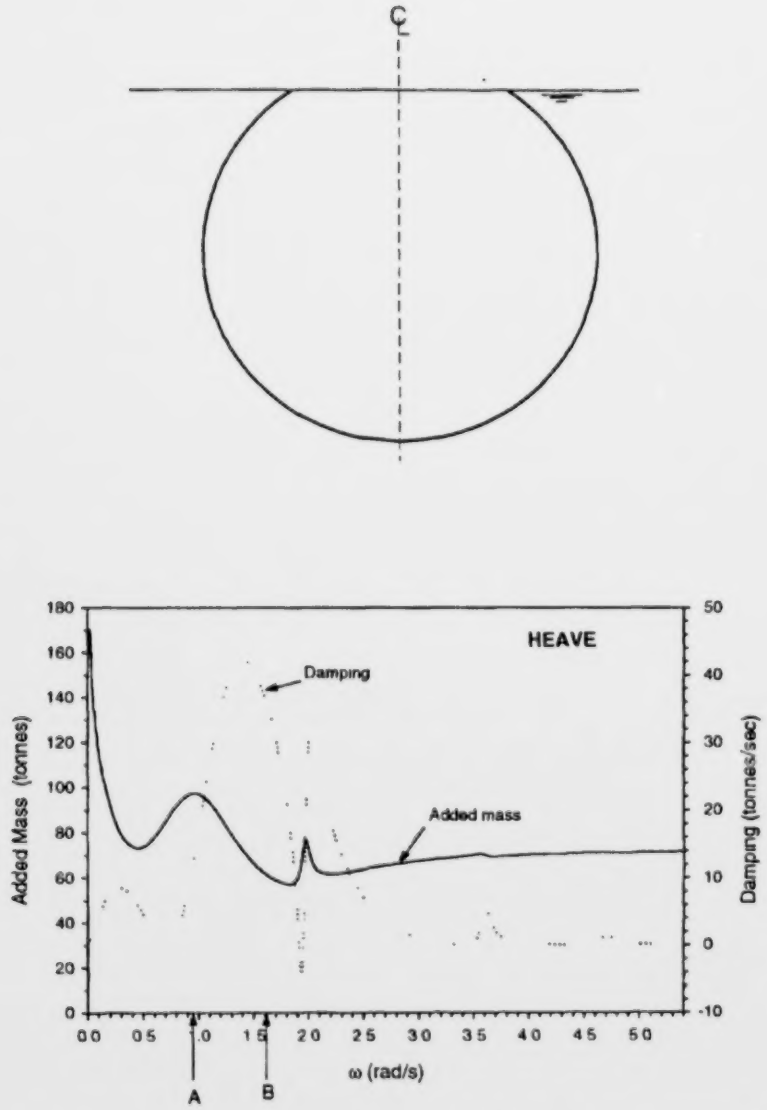

Figure 9. Sectional contour of a surface-piercing circular section (top) and heave added mass and damping (bottom) for the section computed by a 2D source-distribution method. B x H = 12.0 m x 11.0 m. The arrows A (ω = 0.944 rad/s) and B (ω =1.607 rad/s) indicate lower bounds of irregular frequencies predicted by John [1] and the present method, respectively.

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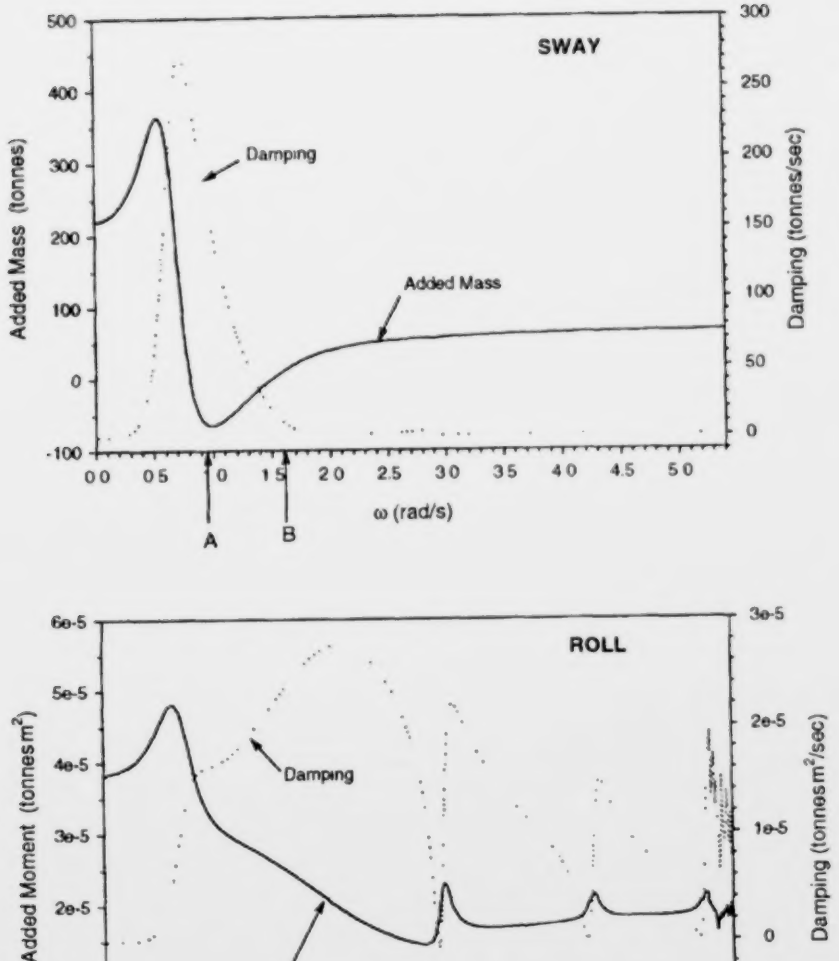

Figure 10. *Sway added mass and damping (top) and roll added moment and damping (bottom) for the amidship section of figure 9 computed by a 2D source-distribution method. The lower bounds of irregular frequencies predicted by John [1] and the present method are indicated by A (ω = 0.944 rad/s) and B (ω = 1.607 rad/s), respectively.*

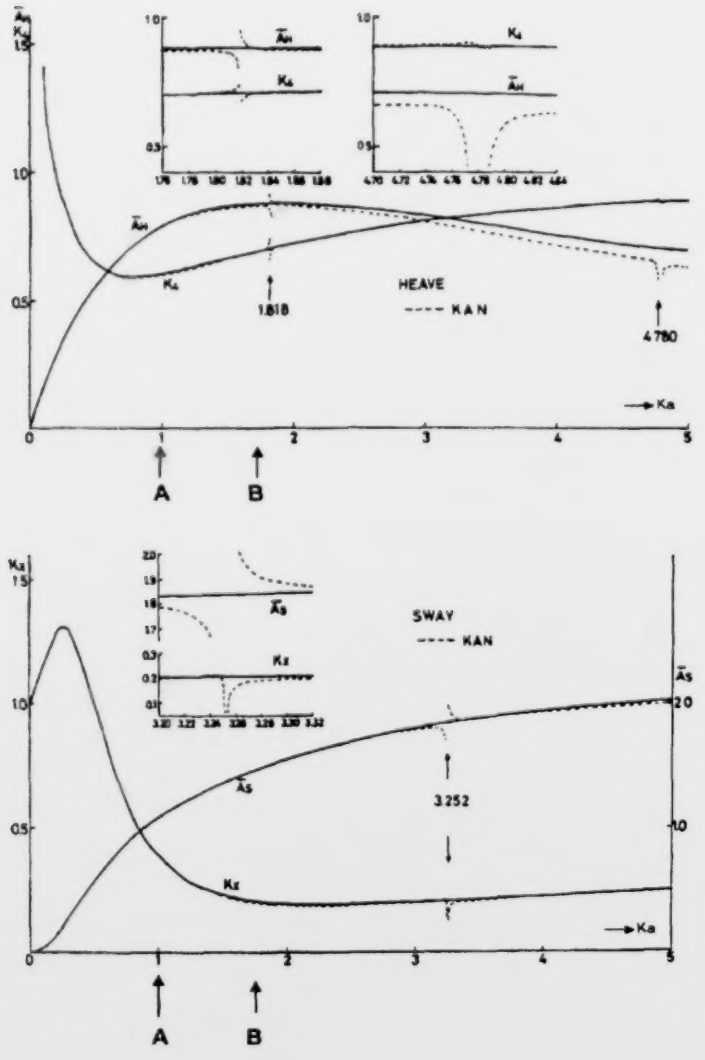

***Figure 11**. Added-mass and damping coefficients for heave (top) and sway (bottom) of a semi-circular cylinder predicted by the ordinary source-distribution method without "lid" (dotted line, calculated by Kan) and by the "lid method" (solid line, calculated by Ohmatsu) The plots are reproduced from figures 8 and 9 of [9], where K_4 and $\bar{A}_H$ are nondimensional added mass and damping coefficients for heave, and K_x and $\bar{A}_S$ are nondimensional added mass and damping coefficients for sway. Here K is wavenumber (k in the present notation) and a is the radius of the semi-circle. The arrows A (Ka = 1.000) and B (Ka = 1.713) indicate lower bounds of irregular frequencies predicted by John [1] and the present method, respectively.*

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References

1. John F. On the motion of floating bodies II. *Comm. on Pure and Appl. Math*, 1950, 3: 45-101
2. Ursell F. Irregular frequencies and the motion of floating bodies. *J. Fluid Mech* 1981, 105: 143-156.
3. Murphy J. Integral equation failure in wave calculations. *J. Waterway, Port, Coastal and Ocean Div.*, Proc. ASCE, 1978; 103 (WW4): 330-334.
4. Wehausen JV. Methods for boundary-value problems in free-surface flows. 3rd David Taylor Lecture, DTNSRDC Report 4622, September 1974.
5. Wu X-J, Price WG. An equivalent box approximation to predict irregular frequencies in arbitrarily-shaped three-dimensional marine structures. *Appl. Ocean Res*, 1986; 8(4): 223-231.
6. Sayer P, Ursell F. Integral-equation methods for calculating the virtual mass in water of finite depth. *2nd Int'l Conf. on Numerical Ship Hydrodynamics*, Berkeley, Calif., 1977.
7. Sayer P. An integral-equation method for determining the fluid motion due to a cylinder heaving on water of finite depth. *Proc. R. Soc. Lond. A*, 1980; 372: 93-110.
8. Rektorys K. *Variational Methods in Mathematics, Science, and Engineering*. Dordrecht; Reidel Pub, 1977.
9. Ohmatsu S. On the Irregular Frequencies in the Theory of Oscillating Bodies in a Free Surface. Paper No. 48, Ship Research Institute, Tokyo, Japan, 1975.
10. Ohmatsu S. A new simple method to eliminate the irregular frequencies in the theory of water wave radiation problems. Paper No. 70, Ship Research Institute, Tokyo, Japan, 1983.
11. Ogilvie TF, Shin YS. Integral equation solutions for time-dependent free-surface problems. *J. Soc. Naval Arch.* Japan, 1978; 143: 41-51.
12. Martin PA. On the null-field equations for water-wave radiation problems. *J. Fluid Mech.* 1981; 113: 315-332.
13. Lee C-H, Sclavounos PD. Removing the irregular frequencies from integral equations in wave-body interactions. *J. Fluid Mech.* 1989; 207: 393-418.
14. Wehausen JV and Laitone EV. Surface Wave. In: Flügge S, editor. *Encyclopedia of Physics*, Vol. 9, Berlin: Springer-Verlag, 1960, Sec. 23.
15. Courant R, Hilbert, D. *Methods of Mathematical Physics*, Vol.1. New York: Interscience, 1953, Ch. 6.
16. Morse PM, Fishbach H. *Methods of Theoretical Physics*, Pt. 2. New York: McGraw-Hill, 1953, p.1148.
17. Troesch BA. Free oscillations of fluid in a container. In: Langer RE, editor. *Boundary Problems in Differential Equations*. U of Wisconsin Press, 1960, pp.279-299.
18. Ando S. Irregular-frequency-free calculations of added masses and damping of two-dimensional cylinders: Examples. DREA Technical Memorandum TM 98/230, October 1998.

Annex A: Method of Wu and Price for approximating irregular frequencies

The method developed by Wu and Price [5] for approximate prediction of irregular frequencies of arbitrarily-shaped 3D bodies is summarized below.

Required input data are:

A_w	waterplane area
A_m	sectional submerged area at amidships
A_c	submerged centreplane area
L	length of ship
B	beam of ship
h	draft of ship
∇	displaced volume

The estimated irregular frequencies are:

$$\omega_{pm} = \sqrt{gk \coth kh_e} \tag{A.1}$$

where $p = 1, 2, 3, \ldots$ and $m = 1, 2, 3, \ldots$ are running indices. The variables k and h_e in (A.1) are defined by,

$$k = \pi \sqrt{\left(\frac{p}{L_e}\right)^2 + \left(\frac{m}{B_e}\right)^2}$$

where L_e, B_e, and h_e are the so-called equivalent length, beam, and draft of an equivalent box, which are defined as follows:

$$\begin{aligned}
L_e &= \beta_1 (\beta_0)^{\alpha_1} L, \\
B_e &= \beta_1 (\beta_0)^{\alpha_2} B, \\
h_e &= \nabla / (L_e B_e), \\
\alpha_1 &= L^2 / (L^2 + B^2) \\
\alpha_2 &= B^2 / (L^2 + B^2) \\
\beta_0 &= (C_w)^{\zeta_0} \\
\beta_1 &= (C_c)^{\zeta_1} \\
\beta_0 &= (C_m)^{\zeta_2}
\end{aligned} \tag{A.2}$$

where

$$C_w = \frac{A_w}{LB}$$

$$C_m = \frac{A_m}{Bh}$$

$$C_v = \frac{A_v}{Lh}$$

and

$$\begin{aligned} c_0 &= 3/4 \\ c_1 &= \left(\frac{1}{8}\right)\left[1 + 6\left|\frac{L-B}{L+B}\right| \ln p\right] \\ c_2 &= \left(\frac{1}{8}\right)\left[1 + 6\left|\frac{L-B}{L+B}\right| \ln m\right]. \end{aligned} \tag{A.3}$$

Equations (A.1), (A.2), and (A.3) are equations (7), (8), and (9) of Wu and Price [5], respectively.

Applying the supplementary condition that "at an irregular frequency a singular behaviour may not occur in a mode of body motion which is of opposite symmetry to the interior potential solution," Wu and Price state that, of the irregular frequencies given by (A.1), ω_{11} and ω_{31} have influence on heave coefficients while ω_{12} and ω_{14} affect sway and roll coefficients.

Annex B: Contour plot of eigenfunctions for a rectangle

By applying the separation of variables method to the boundary-value problem (32), the eigenfunctions W_n corresponding to the eigenvalues k_n, $n = 1, 2, \ldots (k_n = \omega_n / g)$ for a rectangular cylinder of breadth B and draft H can be expressed as (see, e.g., Ando [18]),

$$W_n(x^*, y^*) = \sin n\pi x^* \left(\cosh n\pi x^* + \coth n\pi H^* \sinh n\pi y^*\right), \; n = 1, 2, \ldots$$

where the length scale has been nondimensionalized relative to breadth B, so $x^* = x/B$, $y^* = y/B$, and $H^* = H/B$. To visualize the shapes of eigenfunctions, a simple case of a rectangle with dimensions $B = 2H$ are shown below.

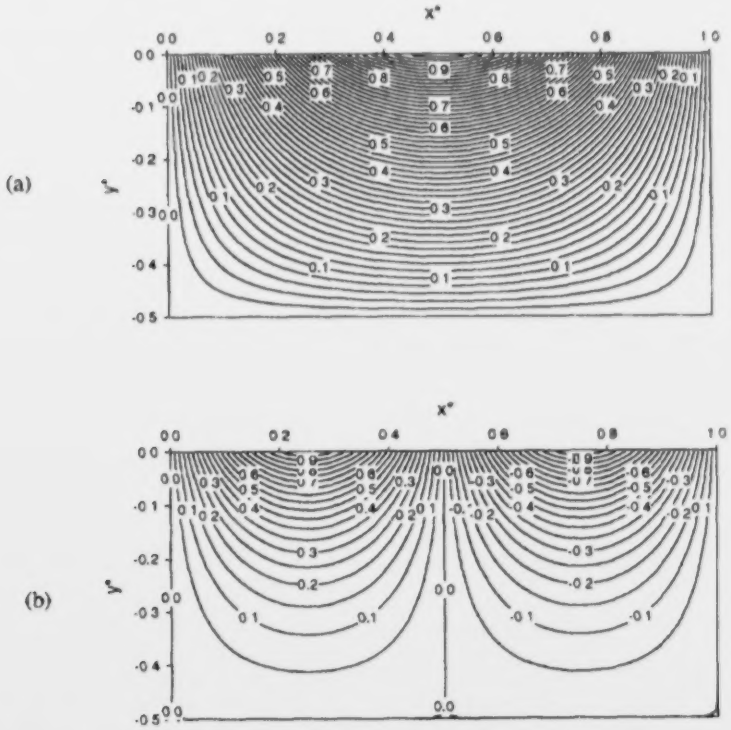

Figure B1. Countour plots of eigenfunctions for rectangles with B=2H: (a) $W_1(x^*, y^*)$ corresponding to fundamental irregular wavenumber k_1 and (b) $W_2(x^*, y^*)$ corresponding to second irregular wavenumber k_2. The numbers in the figures represent the values of eigenfunctions.

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List of symbols/abbreviations/acronyms/initialisms

A, A', A''	mean waterplanes
a_{lm}	2D added-mass coefficient for force in l-mode due to acceleration in m-mode
B	maximum breadth of below calm-water line of floating body
BVP	boundary-value problem
b_{lm}	damping coefficient for force in l-mode due to velocity in m-mode
D, D'	regions inside body below A
E	regions inside S but outside S'
G	Green's function
g	gravitational acceleration
glb	greatest lower bound
H	draft
$\mathfrak{I}$	identity operator
K	class of admissible functions
$\mathfrak{K}_0$	a linear functional transforming functions on S to functions on S
k, k', k''	wavenumbers
k_1, k_1', k_1''	wavenumbers corresponding to fundamental irregular frequencies
L	length between perpendiculars
$\mathfrak{M}_0$	a linear functional transforming functions on S to functions on S
$\mathfrak{N}$	normal-derivative operator
$\mathbf{n}$	unit normal vector to wetted surface pointing into body
n_1, n_2, n_3	x-, y, z-components of $\mathbf{n}$
R, R_1	2D distance
r, r_1	3D distance
S	mean wetted surface
U	mean forward speed of ship
Δ	Laplacian operator
Γ	hull contour at calm-water waterline
η_m	vertical displacement by motion in m mode
λ_{22}	nondimensional damping coefficient for heave
κ_1, κ_y	factors in expressions for k_1
μ_{22}	nondimensional added-mass coefficient for heave
ϕ	velocity potential function representing any of ϕ_m
ϕ_m	velocity potential function for motion in m mode
ρ	mass density of fluid
σ	source strength
ω	oscillation frequency in rad/s
∇	gradient operator

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Lower bounds to irregular frequencies in wave-ship interactions for engineering applications

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ANDO, Samon

5 DATE OF PUBLICATION (month and year of publication of document)	6a NO OF PAGES (total containing information. Include Annexes, Appendices, etc)	6b NO OF REFS (total cited in document)
January 2002	vi, 30	18

7 DESCRIPTIVE NOTES (the category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered)

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An analytical method has been developed to separate the frequency domain for calculations of ship motions and wave loads into two regimes. One is free of irregular frequencies and for which the well-established and well-validated ordinary integral equation method is appropriate. The other includes irregular frequencies and for which an alternative method for suppressing irregular frequencies is required. This dichotomy of the frequency range was achieved by obtaining sharp estimates of lower bounds to the irregular frequencies inherent to the integral-equation method. This work was motivated by the practical need to ensure the efficiency as well as accuracy of the calculations of ship motions and wave loads. Knowledge of two frequency regimes enables one to incorporate a "switch" in the computer codes so as to activate any one of the time-consuming, and often numerically unstable, alternative methods for suppressing irregular frequencies only when it is truly needed. In most cases, however, the need for the latter method would arise only rarely. Theoretically, the present method is based on the monotone relation between the magnitudes of the fundamental eigenvalue and the domain of the interior boundary-value problem associated with the given exterior problem for wave-body interactions, which is derived in this report.

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wave loads
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